Probability and Random Processes

ECS 315

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1 Probability and You

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Everything we do, everything that happens around us, obeys the laws of **probability**.

We can no more escape them than we can escape gravity… "Probability," a philosopher (Bishop Butler) once said, "is the **very guide of life**."

We are all gamblers who go through life making countless bets on the outcome of countless actions.

[Gardner, 1986]

Life is random

In 2005, this statement (which is true)

Life is random

was on display all over the world…

Life is random

[https://www.youtube.com/watch?v=0d8aVzQY4Zo]


Life is random.
Enjoy uncertainty.
Life is random.
Give chance a chance.
Life is random.
Give chance a chance.
Life is random.
Meet iPod shuffle.
The unpredictable new member of the iPod family.
240 songs. A million different ways.
From $99. Mac + PC.
iPod shuffle
Enjoy uncertainty.

iPod shuffle

Life is random.

Applications of Probability Theory

- The subject of probability can be traced back to the 17th century when it arose out of the study of **gambling games**.
- The range of applications extends beyond games into business decisions, insurance, law, **medical tests**, and the social sciences.
- The **stock market**, "the largest casino in the world," cannot do without it.
- The **telephone network**, call centers, and airline companies with their randomly fluctuating loads could not have been economically designed without probability theory.

My Color Scheme for Highlighting

New Term

Definition

Definition 2.13. A ***singleton*** is a set with exactly one element.

Important/Useful Properties/Results

$$\mathbb{E}\left[\sum_{k=1}^{n} c_k g_k(X)\right] = \sum_{k=1}^{n} c_k \mathbb{E}\left[g_k(X)\right].$$

Assumption or goal

- Assumptions: When the
  ng (1
  - Finite Ω: The numb
  - Equipossibility: Th
    occurrence.

Example

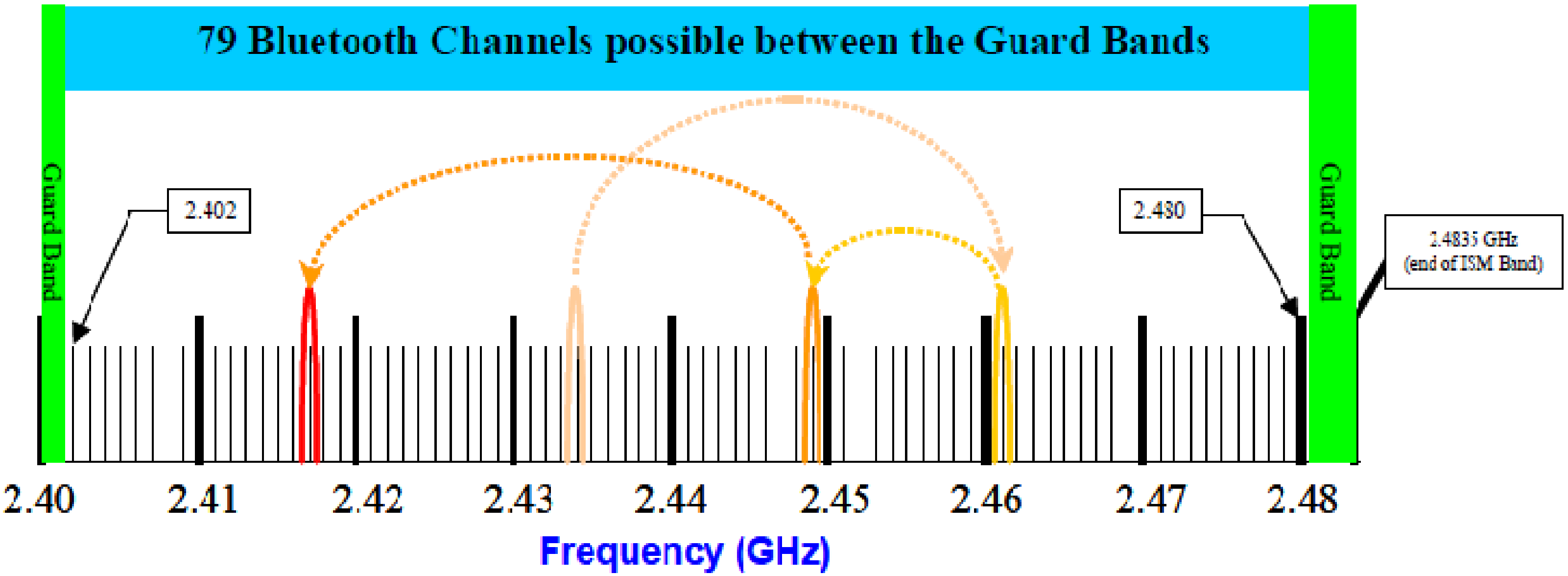
Example 1.1. Some examples from daily life
calculations are involved are the determination
ums, the introduction of new medications on t
polls, weather forecasts, and DNA evidence ir

Miscellaneous

- Histogram is flat ove

FHSS Example: Bluetooth

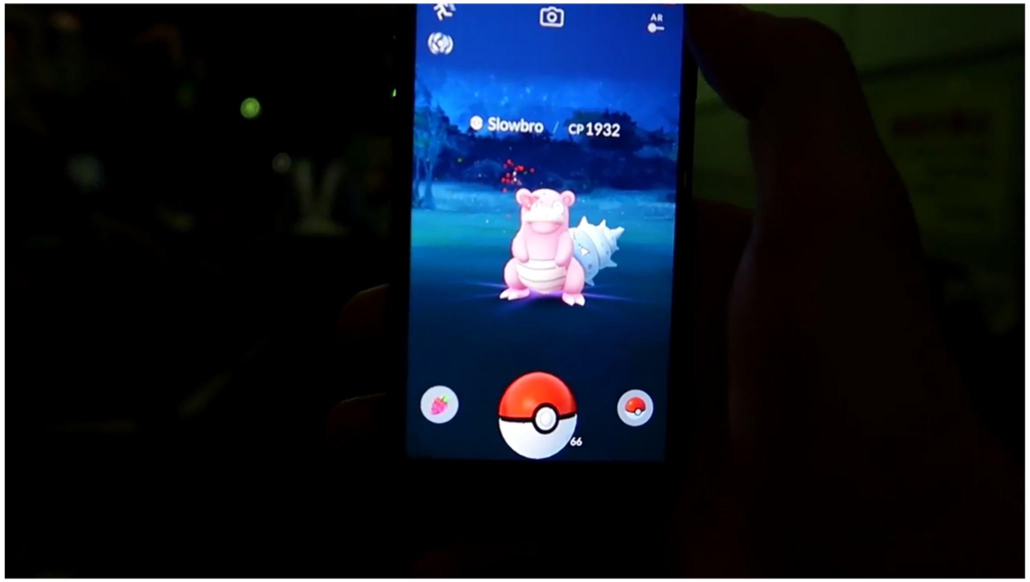
- The band at 2.4 GHz is divided into 79 channels.
- A Bluetooth device, hops frequency at a rate of 1600 hops per second, randomly selecting a channel of 1 MHz to operate.



[https://sites.google.com/site/nearcommunications/adaptative-frequency-hopping]

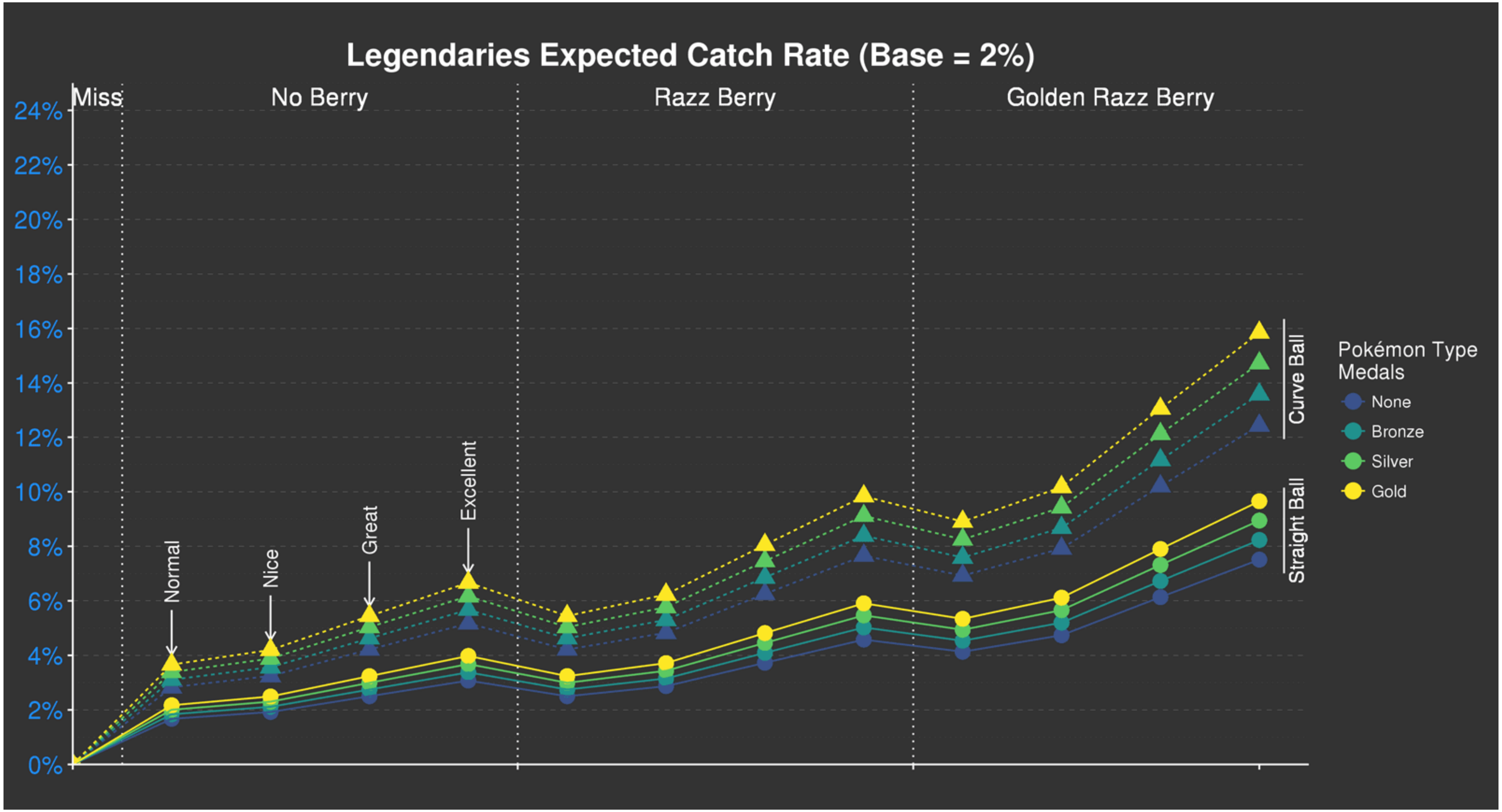
Pokémon GO

- When you throw your Poké Ball to catch a Pokémon, there is this probability of catching (**catch rate**) which depends on the **type of ball** that you use, how you throw the Poké Ball, and whether you use some **special berry**. 
- Each time a Pokemon escapes from a ball, it has a chance of fleeing (**flee rate**) the encounter in a puff of smoke.



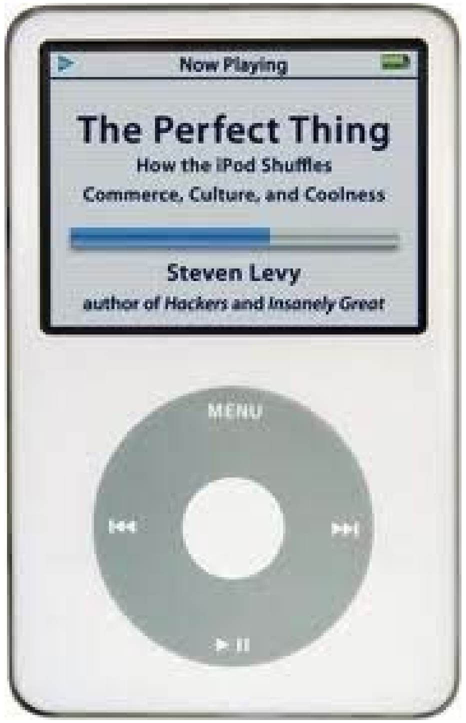
[https://www.youtube.com/watch?v=6VOl8Hm4tG0]

Pokémon GO



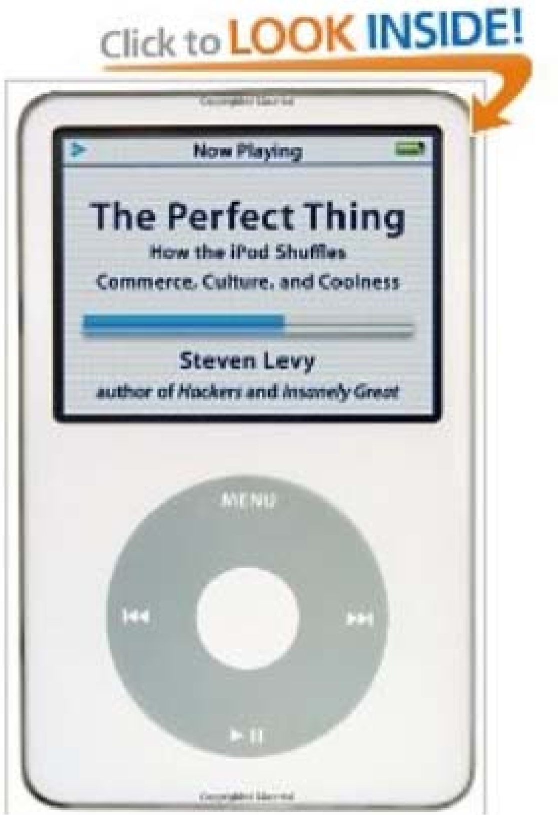
[https://pokemongohub.net/gym-raid-update/legendary-raid-battles/catch-rates/]

"The Perfect Thing"



What is this?

“The Perfect Thing”



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The Perfect Thing: How the iPod Shuffles Commerce, Culture, and Coolness [Hardcover]

Steven Levy (Author)

(27 customer reviews) | Like (1)

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Formats	Amazon Price	New from	Used from
Kindle Edition	--	$10.99	--
Hardcover, Bargain Price	$10.00	$10.00	$3.94
Hardcover, October 24, 2006	--	$0.95	$0.01
Paperback	$14.00	$0.77	$0.01
Audible Audio Edition, Abridged	$17.95	or Free with Audible 30-day free trial	

"The Perfect Thing"

"To me the signature aspect of the iPod is the shuffle… It's a new way to listen to music. Not only that, it seems to be the **symbol for the digital age** itself because, when we go on the internet, really what we're doing is we're shuffling what normally was dealt to us in a pretty rigid hand. When we read news on the internet, we shuffle different publications. When we shuffle our shopping, instead of being physically locked to wherever we do our buying, we could go from one location in the world to somewhere else in a different continent within seconds. To me the shuffle was not only an interesting phenomenon of the iPod itself but it really stood for basically the way we consume media in this age."

[https://www.stevenlevy.com/index.php/about]

What about the shuffle function?

About.com
Part of The New York Times Company
Electronics >
iPhone / iPod
iPhone / iPod | New to iPhone? | Apps | iPho

Is iTunes' Shuffle Mode Truly Random?

By Sam Costello, About.com Guide

http://ipod.about.com/od/advanceditunesuse/a/itunes-random.htm

c|net CNET Australia
Mobile Phones | Computers | Cameras | TVs

MP3 Players > News > iTunes: Just how random is random?

iTunes: Just how random is random?

By David Braue | March 8, 2007 | 127

http://www.cnet.com.au/itunes-just-how-random-is-random-339274094.htm

howstuffworks
Adventure | Auto | Culture | Entertainment | Home & Garden
Computer | Electronics

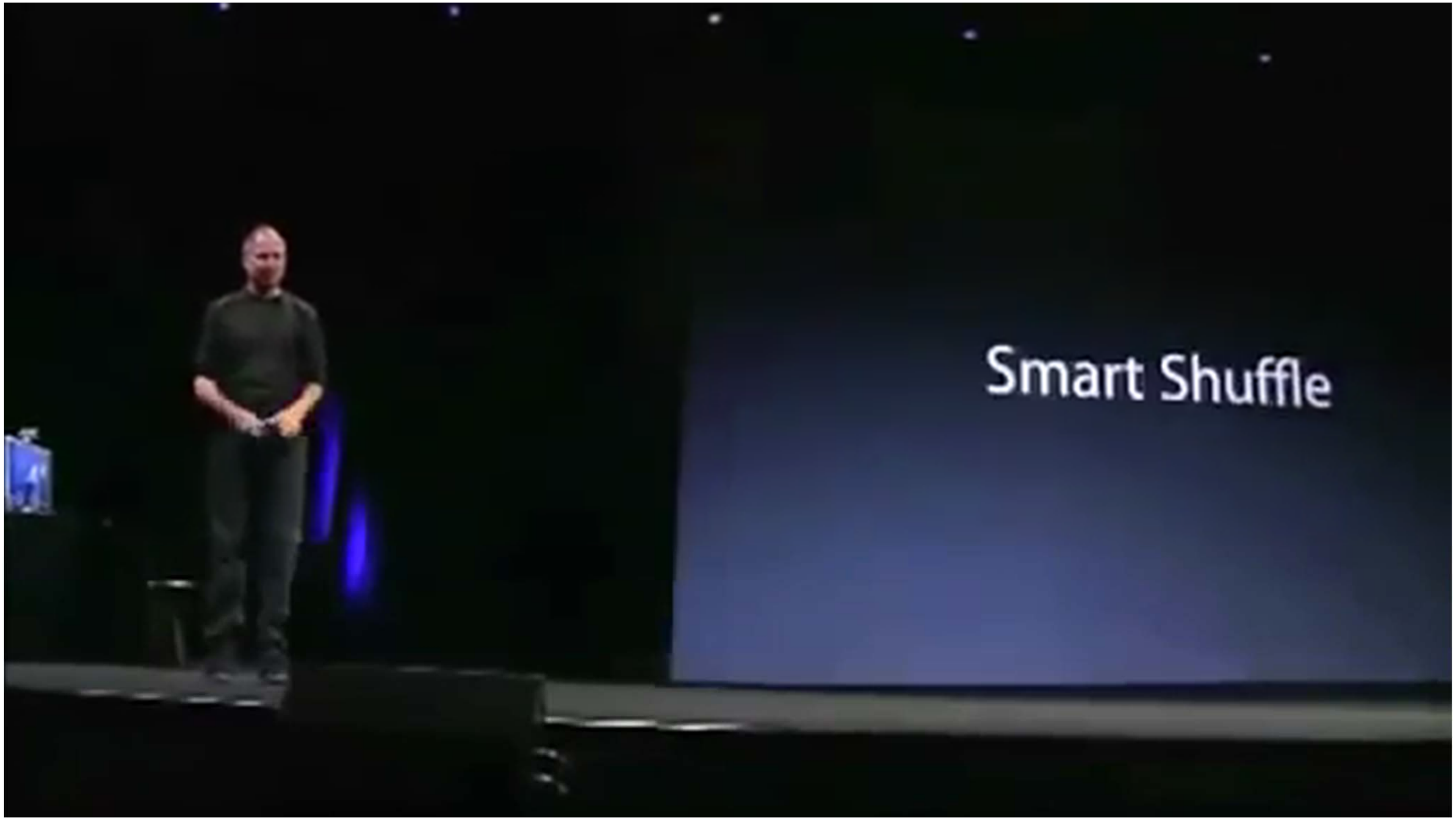
iPod Shuffle Problems: How Random is the iPod Shuffle?

So just how random is the shuffle capability on an iPod Shuffle? Even before the device debuted in 2005, people have wondered about the shuffle function on iPods. Many complain that what they hear from

http://electronics.howstuffworks.com/ipod-shuffle2.htm

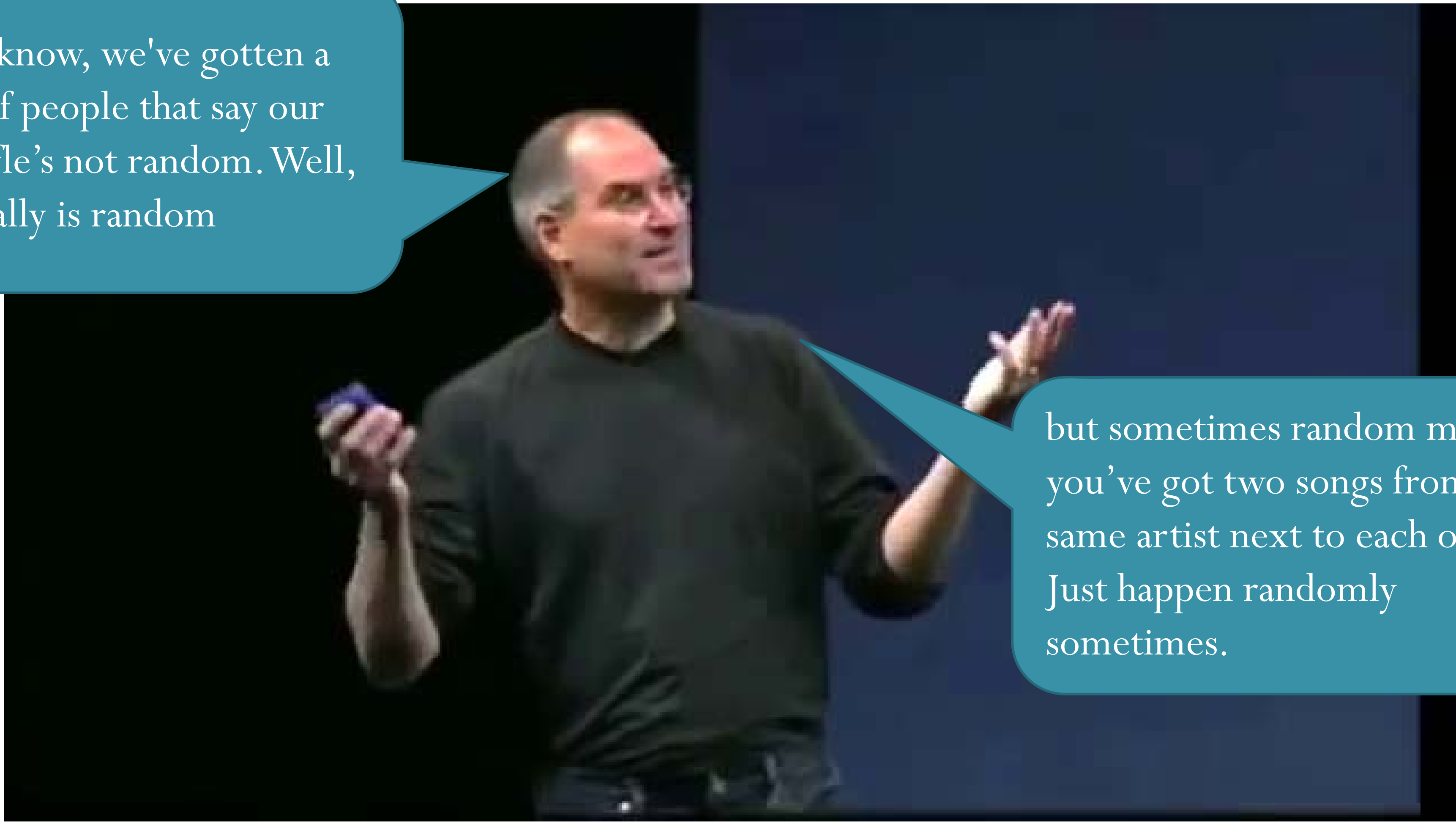
[Example 1.9]

Apple's Smart Shuffle [2005]



[https://www.youtube.com/watch?time_continue=733&v=lg188Ebas9E]

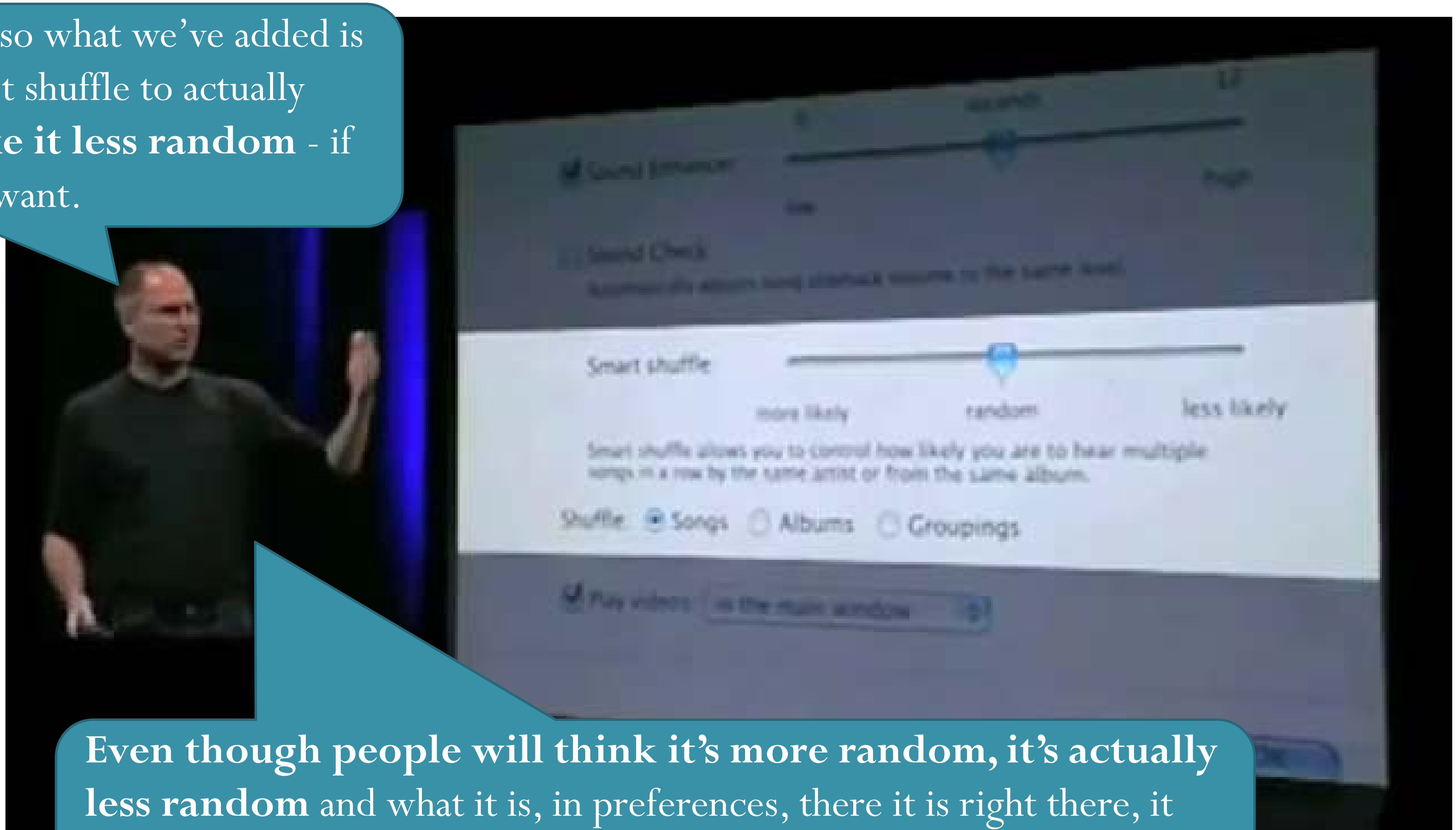
Apple's Smart Shuffle [2005]



Apple’s Smart Shuffle [2005]

‘

And so what we’ve added is smart shuffle to actually **make it less random** - if you want.



Even though people will think it’s more random, it’s actually less random and what it is, in preferences, there it is right there, it says smart shuffle allows you to control how likely you are to hear multiple songs by the same artist or from the same album in a row.

Spotify Labs

How to shuffle songs?

Posted on February 28, 2014 by Lukáš Poláček

Like 1.2K people like this. Be the first of your friends.

At Spotify we take user feedback seriously. We noticed some users complaining about our shuffling algorithm playing a few songs from the same artist right after each other. The users were asking "Why isn't your shuffling random?". We responded "Hey! Our shuffling is random!"

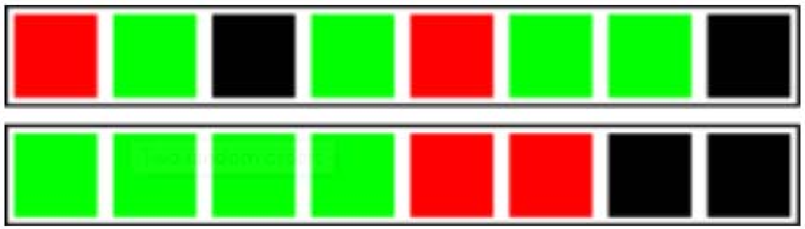
So who was right? As it turns out, both we and the users were right but it's a bit more complicated than that. It also tells a nice story about how to interpret users' feedback.

[https://labs.spotify.com/2014/02/28/how-to-shuffle-songs/]

Spotify Labs

Our perspective

Since the Spotify service launched, we used Fisher-Yates shuffle to generate a *perfectly random shuffling* of a playlist. However, perfectly random means that the following two orders are equally likely to occur (different colors represent different artists):



Gambler's fallacy

At first we didn't understand what the users were trying to tell us by saying that the shuffling is not random, but then we read the comments more carefully and noticed that some people don't want the same artist playing two or three times within a short time period.

[https://labs.spotify.com/2014/02/28/how-to-shuffle-songs/]

USA Currency Coins

- Penny = 1 cent
  (Abraham Lincoln)

- Nickel = 5 cents
  (Thomas Jefferson)

- Dime = 10 cents
  (Franklin D. Roosevelt)

- Quarter = 25 cents
  (George Washington)

Thai Coins

`randi` function

- Generate uniformly distributed **pseudorandom integers**
- `randi(imax)` returns a scalar value between 1 and `imax`.
- `randi(imax,m,n)` and `randi(imax,[m,n])` return an *m*-by-*n* matrix containing pseudorandom integer values drawn from the discrete uniform distribution on the interval [1,`imax`].
  - `randi(imax)` is the same as `randi(imax,1)`.
- `randi([imin,imax],...)` returns an array containing integer values drawn from the discrete uniform distribution on the interval [`imin`,`imax`].

`randi` function: examples

Coin Tosses:

```
>> randi([0,1])
ans =        T, H
     0
>> randi([0,1],10,2)
ans =
     1     0
     1     0
     1     0
     1     1
     1     1
     0     0
     1     1
     0     0
     1     0
     0     0
```

Dice Rolls

```
>> randi([1,6])
ans =
     5
>> randi([1,6],10,2)
ans =
     5     1
     2     1
     3     3
     3     6
     4     3
     5     4
     5     2
     2     5
     5     2
     4     4
```

`randi` function: examples

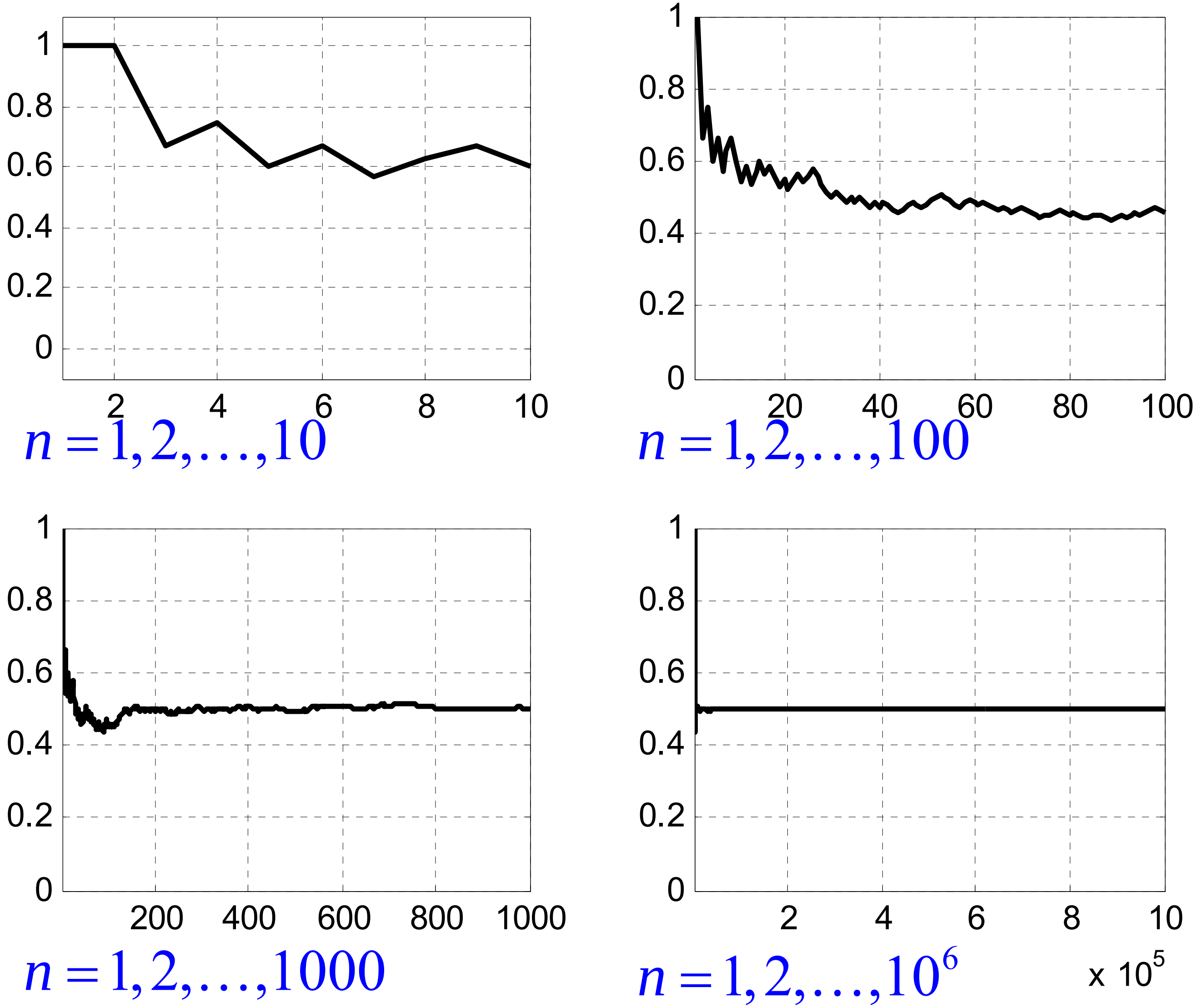
Coin Tosses:

```
>> S = ['T','H']
S =
TH
>> S(randi([1,2]))
ans =
H
>> S(randi([1,2],10,2))
ans =
TT
HH
HT
TT
HT
TT
TH
HT
HH
HT
```

Coin Tossing: Relative Frequency

$$\frac{N(A,n)}{n}$$

If a fair coin is flipped a large number of times, the **proportion** of heads will tend to get closer to 1/2 as the number of tosses increases.



$n = 1, 2, \ldots, 10$

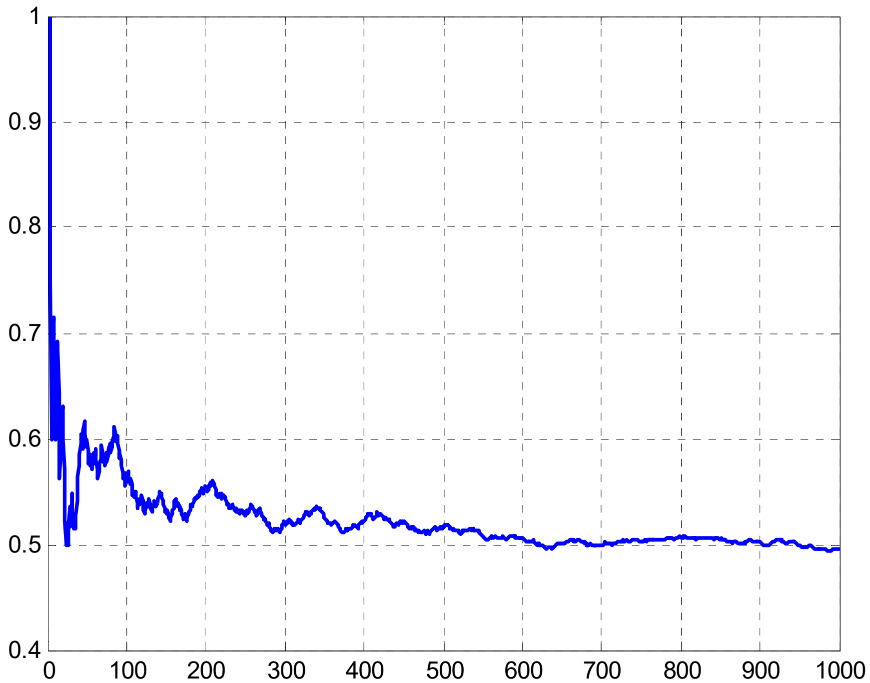
$n = 1, 2, \ldots, 100$

$n = 1, 2, \ldots, 1000$

$n = 1, 2, \ldots, 10^6$

Coin Tossing: Relative Frequency

```
close all; clear all;
N = 1e3; % Number of trials (number of times that the coin is tossed)
s = randi([0,1],1,N);   % Generate a sequence of N Coin Tosses.
                        % The results are saved in a row vector s.
NH = cumsum(s); % Count the number of heads
plot(NH./(1:N),'LineWidth',1.5); grid on % Plot the relative frequencies
```

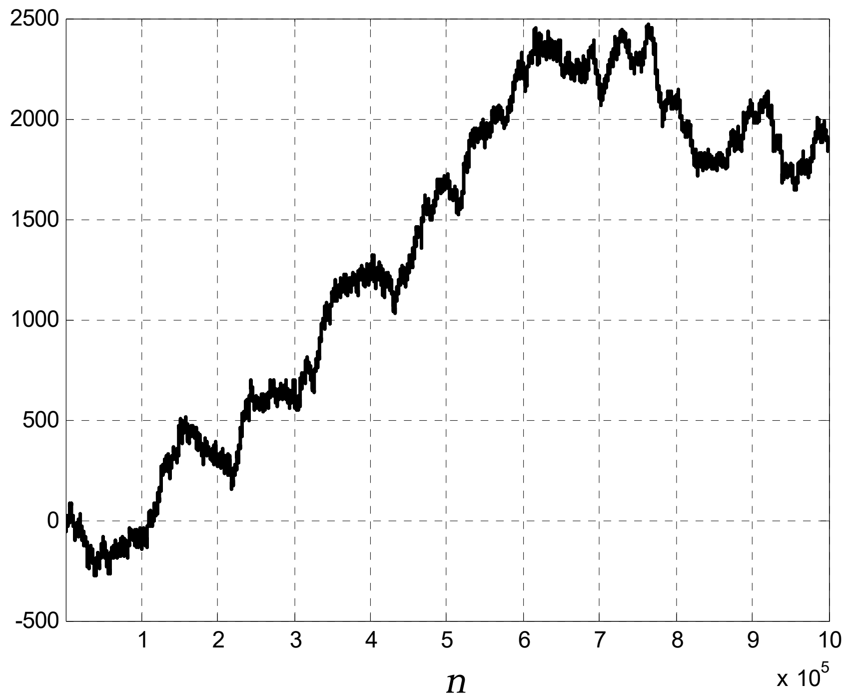


Coin Tossing: Relative Freq. vs. #H-#T

This statement does not say that the difference between #H and #T will be close to 0.

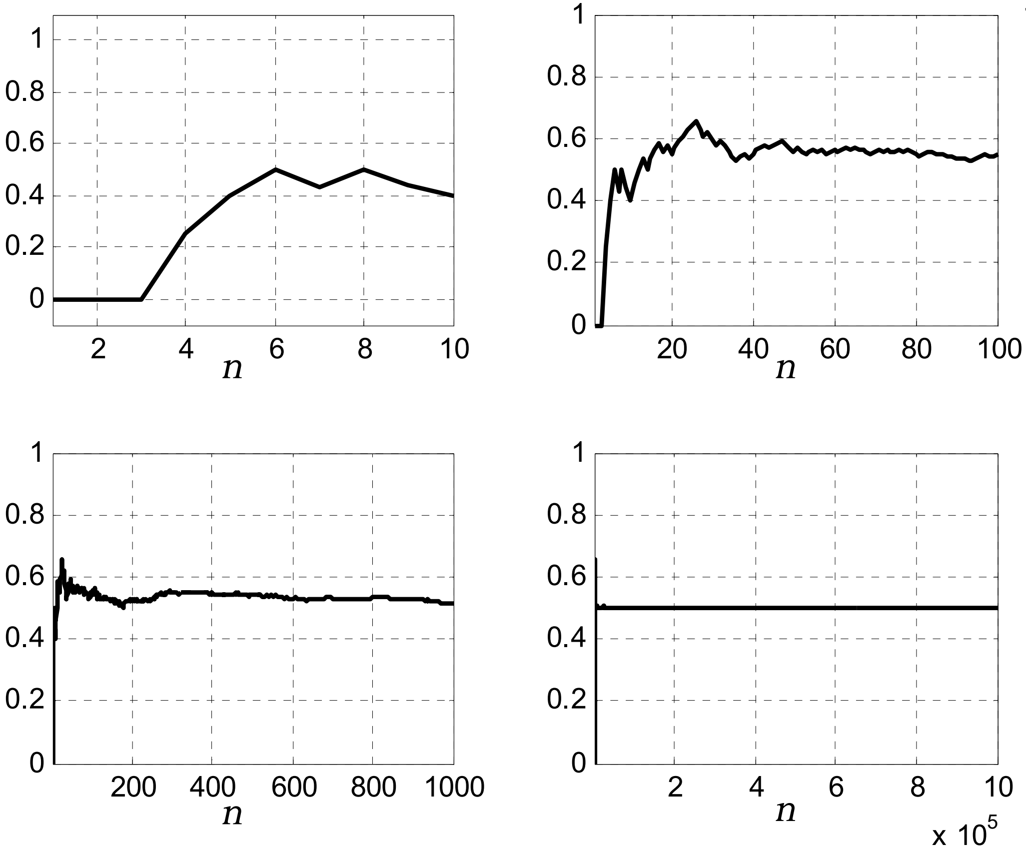
If a fair coin is flipped a large number of times, the **proportion** of heads will tend to get closer to 1/2 as the number of tosses increases.

The **difference** between #H and #T will **not** converge to 0.



Another Experiment

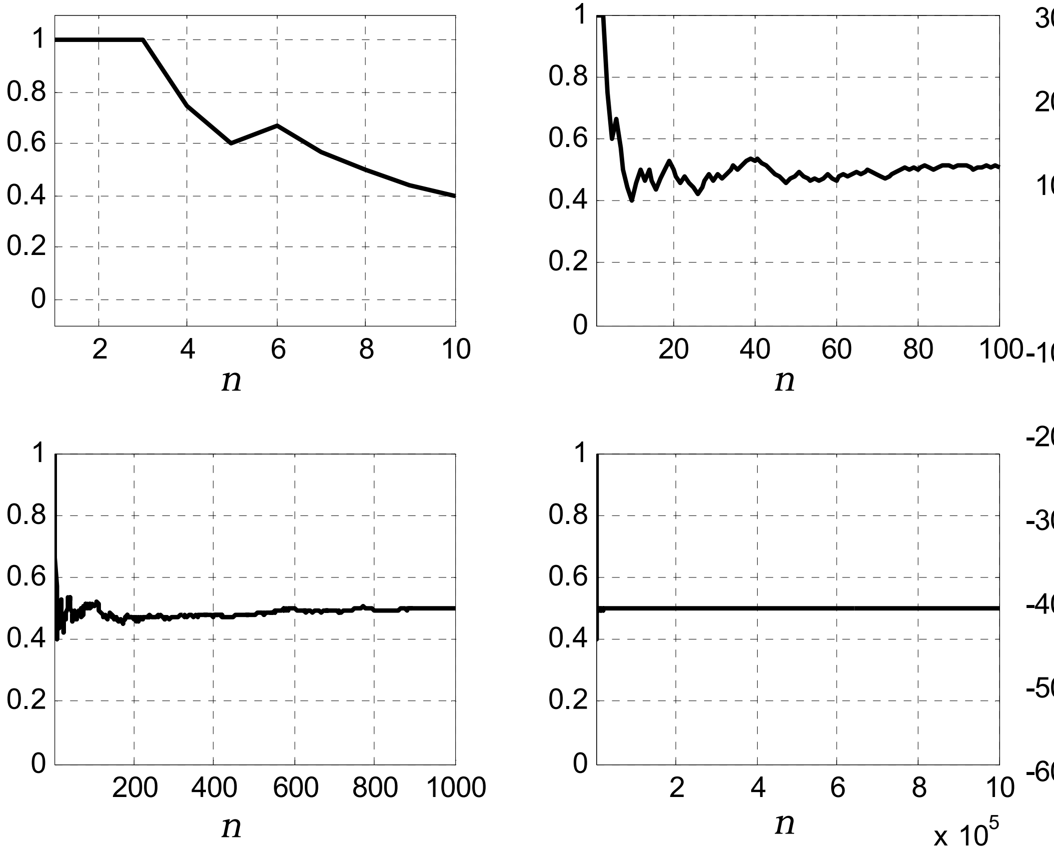
Relative Freq.



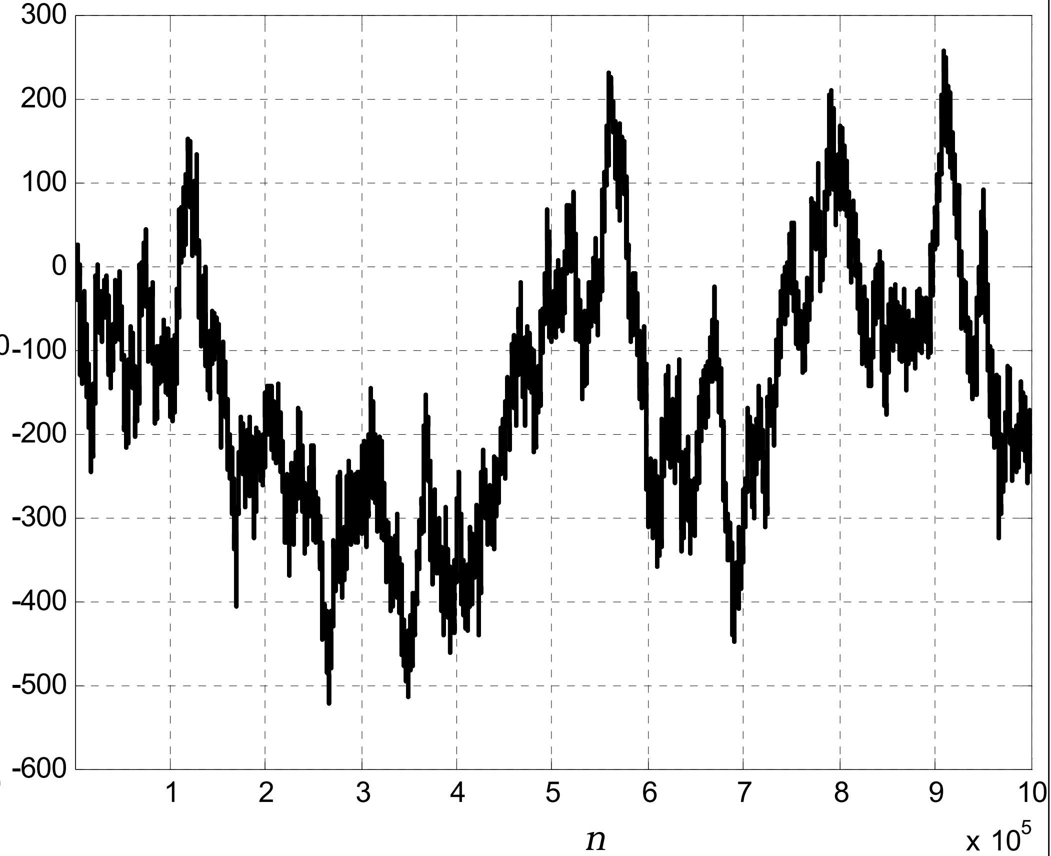
#H-#T

Another Experiment

Relative Freq.



#H-#T



Summary:

- Ingredient of Probability Theory:
  - Random experiment
  - Outcome ω ——— each outcome represent a result from the experiment
  - Sample space Ω —collection (set) of all possible outcomes
  - Event A ———— collection of outcomes that meets some specifications

    $(\subset \Omega)$

    └ define outcomes of interest from a random experiment
  - $P(A)$ = probability of event A
    - For a random experiment and a specific event A,
      when the experiment has been performed,
      the event may occur or not occur.
      The probability that it occurs is denoted by $P(A)$.

Summary:

- Q: How do we interpret the value of probability?

  What does the value of $P(A)$ tells us about event A?
- A: "long-run frequency interpretation"
  - Repeat the experiment n times (n should be large).
  - Count the "fraction of times that A occurs" among these n repetitions.

    This is called the "**relative frequency**" of event A.
  - **Law of Large Numbers (LLN)**
    - As $n \to \infty$, the relative frequency of event A will converge to $P(A)$.
    - When n is not ∞, but large, the fraction should be close to $P(A)$.